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Hawking Radiation of Charged and Magnetized Particles from the Kerr-Newman-Kasuya Black Hole via Covariant Anomaly

Xiao-Xiong Zeng · Shu-Zheng Yang

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Abstract Hawking radiation of a particle with electric and magnetic charges from the Kerr-Newman-Kasuya black hole is discussed in the dragging coordinate frame via the anomaly cancellation method, initiated by Robinson and Wilczek. We redefine an equivalent charge of the charged and magnetized black hole by reconstructing the electromagnetic field tensor. We adopt the refined covariant anomaly cancellation method to determining the compensating fluxes of charge flow and energy momentum tensor, which are proved to precisely match with those of the 2-dimensional blackbody radiation at the Hawking temperature with an appropriate chemical potential.

Keywords Hawking radiation · Covariant anomaly · Black hole · Magnetic charge

1 Introduction

Hawking radiation is one of the most intriguing phenomenons in black hole physics. Besides the foremost paradigm of Stephen Hawking [1, 2], many derivations have been developed [3–8] subsequently. Recently, Robinson and Wilczek put forward a novel method to derive Hawking radiation from the anomaly point of view [9]. Actually Christiansen and Fulling [10] many years ago have shown that it was sometimes possible to use an anomaly in conformal symmetry to derive important constraints on the energy momentum tensors of quantum fields in a black hole background. However their observation was quite special, and might be regarded an isolated curiosity. Since their work was limit to the 2-dimensional space time and the fields were massless as well as there was no back-scattering effect. Also,

X.-X. Zeng (✉)
Department of Physics and Engineering Technology, Sichuan University of Arts and Science, Dazhou, Sichuan, 635000, People's Republic of China
e-mail: xxzengphysics@163.com

S.-Z. Yang
Institute of Theoretical Physics, China West Normal University, Nanchong, Sichuan, 637002, People's Republic of China

as a conceptual matter, the central role ascribed to conformal symmetry seems rather artificial there. The work of Robinson et al. is very universal, partly because physics near the horizon of any higher dimensional background space time can be described by an infinite collection of 2-dimensional fields via the dimensional reduction technique and partly because their work isn't limit to scalar field. In the 2-dimensional reduction, due to the horizon is a null hyper-surface where ingoing modes don't affect the outside physics of the hole classically but do at the quantum level, the effective field becomes chiral and suffers from gravitational anomaly with respect to the general coordinate symmetry as the horizon-skimming modes are integrated out to construct the effective field. The energy momentum tensor flux correspondingly doesn't conserve at the horizon owning to the ill-defined of the time coordinate. To restore general covariance at the quantum level, a flux therefore must be introduced to cancel gravitational anomaly at the horizon. The result shows that the compensating energy momentum flux has an equivalent form to that of Hawking radiation. Following that work, Iso et al. Subsequently extended this method to the cases of the charged black hole [11] and rotating black hole [12] (others see [13–22]) assimilating azimuthal quantum number m to electric charge e via gauge anomaly and gravitational anomaly successfully. It should be mentioned that both the consistent anomaly and covariant anomaly are needed to determine the compensating flux until the very recent work in [23] was proposed. It was suggested the same work can be done only by adopting the covariant gauge anomaly and gravitational anomaly. Note that the boundary condition should always be covariant. This method, as emphasized in [23], is conceptually clean and mathematically economical, we hence employ the covariant anomaly cancellation conditions to discuss quantum anomaly and Hawking radiation of charged and magnetized particle from the Kerr-Newman-Kasuya black hole. Though the anomaly cancellation method has been extended to various black holes, the charged and magnetized background space time hasn't been treated of. Because in this case, one should discuss not only gauge flux to electric charge but also gauge flux to magnetic charge, which is very trouble. To overcome this problem here, we reconstruct the electromagnetic field tensor of the background space time by regarding the electric and magnetic charges are concentrated on a conducting sphere. For the sake of simplicity, we consider the case that the density ratio of radiant electric and magnetic charges equal to that of the source [24]. In this way, the equivalent charge and gauge potential of the charged and magnetized black hole are introduced. It also should be noted that in the dragging coordinate frame, because the observer is co-rotating with the rotating black hole, which behaves locally like a kind of non-rotating coordinate frame, the $U(1)$ gauge current thus isn't related to the flux of angular momentum but only depends on the equivalent charge. To restore the gauge covariance and general covariance at the quantum level, we hence only need to cancel one gauge anomaly and one gravitational anomaly at the event horizon of the Kerr-Newman-Kasuya black hole in the dragging coordinate frame.

The remainder of this paper is outlined as follows. In Sect. 2, we reconstruct the electromagnetic field tensor to redefine the equivalent charge and gauge potential of the Kerr-Newman-Kasuya black hole. Then we reduce the 4-dimensional space time to an effective 2-dimensional theory in Sect. 3 and study its Hawking radiation by the covariant gauge and gravitational anomalies in Sect. 4. Section 5 is devoted to our concluding remarks.

2 The Equivalent Charge and Gauge Potential of the Charged and Magnetized Background Space Time

For the charged and magnetized black hole, the electric charge and magnetic charge are concentrated on the hole and the outside of the hole is an electromagnetic vacuum, the black hole thus can be treated as a conducting sphere [24]. In this case, the electromagnetic tensor can be defined as [25]

$$F_{\mu\nu} = \nabla_\nu A_\mu - \nabla_\mu A_\nu + G^+_{\mu\nu}, \tag{1}$$

where $G^+_{\mu\nu}$ is the Dirac string term [25]. And the Maxwell equations read off

$$\nabla_\nu F^{\mu\nu} = 4\pi\rho_e u^\mu, \tag{2}$$

$$\nabla_\nu F^{+\mu\nu} = 4\pi\rho_g u^\mu, \tag{3}$$

where $F^{+\mu\nu}$ is the dual tensor of $F^{\mu\nu}$, ρ_e and ρ_g stand for the densities of electric and magnetic charges, while u^μ represents the 4-velocity. If we define a new real anti-symmetric tensor

$$\tilde{F}^{\mu\nu} = F^{\mu\nu}\cos\beta + F^{+\mu\nu}\sin\beta, \tag{4}$$

where β denotes a real angle. Equations (2) and (3) will change as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi(\rho_e\cos\beta + \rho_g\sin\beta)u^\mu, \tag{5}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 4\pi(-\rho_e\sin\beta + \rho_g\cos\beta)u^\mu. \tag{6}$$

Then let

$$\rho_e\cos\beta + \rho_g\sin\beta = \rho_h, \tag{7}$$

$$-\rho_e\sin\beta + \rho_g\cos\beta = 0. \tag{8}$$

Besides yields $\rho_e/\rho_g = \cos\beta$, the Maxwell equations can be simplified as

$$\nabla_\nu \tilde{F}^{\mu\nu} = 4\pi\rho_h u^\mu, \tag{9}$$

$$\nabla_\nu \tilde{F}^{+\mu\nu} = 0, \tag{10}$$

where $\tilde{F}_{\mu\nu} = \nabla_\nu \tilde{A}_\mu - \nabla_\mu \tilde{A}_\nu$, and

$$\tilde{A}_\mu = (\tilde{A}_0, \tilde{A}_1, \tilde{A}_2, \tilde{A}_3), \tag{11}$$

is the corresponding general coordinate. While $J^\mu = \rho_h u^\mu$ is employed, (9) also can be written as

$$\frac{\partial}{\partial x^\nu}(\sqrt{-g}\,\tilde{F}^{\mu\nu}) = 4\pi\sqrt{-g}\,J^\mu. \tag{12}$$

Obviously, (12) is similar to the Maxwell equation corresponding to the source only with electric charge. Therefore, if we consider the black hole as a conducting sphere while the electric charge and the magnetic charge are concentrated on the black hole with the density rate as $\rho_e/\rho_g = \cos\beta$, we have

$$Q_h^2 = Q_e^2 + Q_g^2, \tag{13}$$

where Q_e and Q_g are the electric and magnetic charges of the hole, and Q_h stands for the equivalent charge corresponding to the density ρ_h.

3 Two-Dimensional Theory of the Kerr-Newman-Kasuya Black Hole

The metric of the Kerr-Newman-Kasuya black hole [26, 27] can be written as

$$ds^2 = -\left(1 - \frac{2Mr - (Q_e^2 + Q_g^2)}{\Sigma}\right)dt_K^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$
$$- \frac{2(2Mr - (Q_e^2 + Q_g^2))a\sin^2\theta}{\Sigma}dt\varphi$$
$$+ \left[(r^2 + a^2)\sin^2\theta + \frac{(2Mr - (Q_e^2 + Q_g^2))a^2\sin^4\theta}{\Sigma}\right]d\varphi^2, \tag{14}$$

where $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta' = r^2 + a^2 + Q_e^2 + Q_g^2 - 2Mr$, M, a, Q_e and Q_g represent the mass, angular momentum of unit mass, electric charge and magnetic charge of the black hole respectively. Considering (13), line element (14) can be rewritten as

$$ds^2 = -\left(1 - \frac{2Mr - Q_h^2}{\Sigma}\right)dt_K^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 - \frac{2(2Mr - Q_h^2)a\sin^2\theta}{\Sigma}dt\varphi$$
$$+ \left[(r^2 + a^2)\sin^2\theta + \frac{(2Mr - Q_h^2)a^2\sin^4\theta}{\Sigma}\right]d\varphi^2, \tag{15}$$

where $\Delta = r^2 + a^2 + Q_h^2 - 2Mr$. The inner and outer horizons in this case are given by

$$r_\pm = M \pm (M^2 - a^2 - Q_h^2)^{\frac{1}{2}}. \tag{16}$$

The corresponding nonvanishing component of the electromagnetic vector potential is

$$\tilde{A} = -\frac{Q_h r}{\Sigma}dt + \frac{Q_h r a\sin^2\theta}{\Sigma}d\varphi. \tag{17}$$

Because we would formulate the effective field in the dragging coordinate frame, so the dragging coordinate transformation should be performed firstly. Let

$$\frac{d\varphi}{dt} = -\frac{g_{03}}{g_{33}} = \frac{a(r^2 + a^2 - \Delta)}{(r^2 + a^2)^2 - \Delta a^2\sin^2\theta}, \tag{18}$$

line element (15) would be simplified as

$$ds^2 = \hat{g}_{00}dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2, \tag{19}$$

where

$$\hat{g}_{00} = -\frac{\Sigma\Delta}{(r^2 + a^2)^2 - \Delta a^2\sin^2\theta}. \tag{20}$$

Equation (19) stands for a 3-dimensional hypersurface in the 4-dimensional space time. To formulate the effective field, one should reduce the matter fields, for the sake of simplicity only for scalar field, on this hypersurface to that on a 2-dimensional space time. The action takes the form as

$$S[\varphi] = \frac{1}{2}\int d^D x\sqrt{-g}\varphi\nabla^2\varphi$$
$$= \frac{1}{2}\int dx^3 \sin\theta\phi_{l.m}\left[\left(-\frac{(r^2+a^2)^2}{\Delta} + a^2\sin^2\theta\right)\partial_t^2 + \frac{1}{r^2}\partial_r r^2\Delta\partial_r + \frac{1}{\sin\theta}(\partial_\theta \sin\theta\partial_\theta)\right]\phi_{l.m}. \quad (21)$$

Performing the partial wave decomposition at the event horizon and making the tortoise coordinate transformation defined by [28]

$$\frac{dr_*}{dr} = \sqrt{\frac{-g_{11}}{\hat{g}_{00}}} = f(r)^{-1} = \frac{r^2+a^2}{\Delta}, \quad (22)$$

when the subordinate terms are ignored, we find physics that are near the event horizon can be effectively described by an infinite collection of (1 + 1)-dimensional scalar fields with the action

$$S[\varphi] = \sum_{l,m}\int \Psi dt dr \varphi_{lm}\left[-\frac{1}{f(r)}\partial_t^2 + \partial_r f(r)\partial_r\right]\varphi_{lm}, \quad (23)$$

where $f = \Delta/(r^2+a^2)$ and $\Psi = (r^2+a^2)/2$ is the dilaton background, which is often ignored in the static background. And the electromagnetic gauge potential in the dragging coordinate frame takes the form as

$$\tilde{A}_t = -\frac{Q_h r}{r^2+a^2}. \quad (24)$$

4 Hawking Radiation and Quantum Anomaly

Now, we concentrate on deriving the Hawking radiation from the viewpoint of covariant anomaly. In the reduced 2-dimensional space time, when omitting the classically irrelevant ingoing modes, the effective field becomes chiral and each partial wave suffers from gauge and gravitational anomalies with respect to gauge and general coordinate symmetries respectively. The underlying theory of course is covariant, the gauge and gravitational anomalies therefore should be cancelled at the event horizon. We first discuss the flux of charge flow in the effective field. For convenience, we divide the outside region of the horizon into $[r_+, r_+ + \varepsilon]$ and $[r_+ + \varepsilon, +\infty]$. When the classically irrelevant ingoing modes are integrated out, gauge symmetry breaks out and the near-horizon gauge flux $\tilde{J}^r_{(H)}$ thus follows

$$\nabla_r \tilde{J}^r_{(H)} = \frac{Q_h^2}{4\pi}\tilde{F}^{rt} = \frac{Q_h^2}{2\pi}\partial_r \tilde{A}_t, \quad (25)$$

where we have adopted the covariant anomalous equation [29]

$$\nabla_\mu \tilde{J}^\mu = \pm\frac{e^2}{4\pi\sqrt{-g}}\varepsilon_{\mu\nu}\tilde{F}^{\mu\nu}, \quad (26)$$

in which, $+(-)$ corresponds to the left (right) handed field and $\varepsilon^{\mu\nu}$ is the anti-symmetric tensor ($\varepsilon^{10} = \varepsilon^{01} = -1$). But in this context, the gauge charge is the equivalent charge, the charge e in (26) therefore is substituted by equivalent charge Q_h. While in $[r_+ + \varepsilon, +\infty]$, its partner $\tilde{J}_0^r$ obeys the covariant conservation equation

$$\partial_r \tilde{J}^r_{(0)} = 0. \tag{27}$$

Solving (25) and (27) yield

$$\tilde{J}^r_{(H)} = a_h + \frac{Q_h^2[\tilde{A}_t(r) - \tilde{A}_t(r_+)]}{2\pi}, \tag{28}$$

$$\tilde{J}^r_{(0)} = a_0, \tag{29}$$

where a_h and a_0 are integration constants, which respectively stand for the values of the covariant fluxes of charge flow at infinity and the event horizon.

Since the effective field has been divided into two regions, the gauge flux outside the event horizon can be written as $\tilde{J}^\mu_{(2)} = \tilde{J}^u_{(0)}\Theta_+(r) + \tilde{J}^\mu_{(H)}H(r)$ by the scalar step function $\Theta_+ = \Theta(r - r_+ - \varepsilon)$ and scalar top hat function $H = 1 - \Theta_+$. Then the Ward identity can be expressed as

$$\nabla_\mu \tilde{J}^\mu = \partial_r \tilde{J}^r = \partial_r\left(\frac{Q_h^2}{2\pi}\tilde{A}_t H\right) + \left(\tilde{J}^r_{(0)} - \tilde{J}^r_{(H)} + \frac{Q_h^2 \tilde{A}_t(r)}{2\pi}\right)\delta(r - r_+ - \varepsilon). \tag{30}$$

Taking into account the Wess-Zumino term, one can redefine the covariant energy momentum tensor as an anomaly-free one $\tilde{J}'^r = \tilde{J}^r - Q_h^2\tilde{A}_t H(r)/2\pi$. That is, the first term in the right of (30) shall be cancelled by the quantum effect of the ingoing modes that we have omitted. To hold the underlying gauge invariance at the quantum level, the coefficients of the delta function should vanish. Therefore we have

$$a_0 = a_h - \frac{Q_h^2 \tilde{A}_t(r_+)}{2\pi}. \tag{31}$$

To determine a_0, we impose the covariant boundary conditions, which require the covariant charge current to vanish at the event horizon. The covariant flux of charge flow thus can be written explicitly as

$$a_0 = -\frac{Q_h^2 \tilde{A}_t(r_+)}{2\pi} = \frac{Q_h^3 r_+}{2\pi(r_+^2 + a^2)}. \tag{32}$$

This flux is required to restore the gauge invariance at the quantum level in the dragging coordinate frame and will be shown to precisely equal to that of Hawking radiation from the Kerr-Newman-Kasuya black hole.

Next, we turn to determining the compensating flux of energy momentum tensor. Similarly, when the classically insignificant ingoing modes are integrated out, gravitational anomaly with respect to the general coordinate symmetry exhibits due to the pileup of high frequency modes at the horizon. An energy momentum tensor flux hence must be introduced to save the general covariance. The total flux of energy momentum tensor also can be divided into $\tilde{T}^\mu_\nu = \tilde{T}^u_{t(0)}\Theta_+(r) + \tilde{T}^\mu_{t(H)}H(r)$, which satisfies the covariant gravitational anomaly [30]

$$\nabla_\mu \tilde{T}^\mu_\nu = -\frac{1}{96\pi}\varepsilon_{\mu\nu}\partial^\mu R = \tilde{A}_\mu = \frac{1}{\sqrt{-g}}\partial_\mu \tilde{N}^\mu_\nu. \tag{33}$$

In $[r_+ + \varepsilon, +\infty]$, the covariant flux of energy momentum tensor $\tilde{T}^{u}_{t(0)}$ in the gauge field background obeys the covariant conservation equation

$$\nabla_\mu \tilde{T}^{\mu}_{\nu(0)} = \tilde{F}_{\mu\nu} \tilde{J}^{\mu}_{(0)}. \tag{34}$$

Making use of (29), it can be solved as

$$\tilde{T}^{r}_{t(0)} = d_0 + a_0 \tilde{A}_t(r), \tag{35}$$

in which, d_0 is an integration constant, which stands for the value of the covariant flux of energy momentum tensor at infinity. On the other hand, in the near-horizon region, due to the pileup of the divergent energy momentum tensor, its partner $\tilde{T}^{\mu}_{t(H)}$ satisfies the modified covariant anomalous equation

$$\nabla_\mu \tilde{T}^{\mu}_{\nu(H)} = \tilde{F}_{\mu\nu} \tilde{J}^{\mu}_{(H)} + \tilde{A}_\nu. \tag{36}$$

In the right of (36), the second term stems from the covariant gravitational anomaly for the energy momentum tensor, which yields $\tilde{N}^r_t = [2ff'' - (f')^2]/192\pi$. In the same way, while (28) is employed, the solution to the energy momentum tensor in the region $[r_+, r_+ + \varepsilon]$ is

$$\tilde{T}^{r}_{t(H)} = d_h + \int_{r_+}^{r} dr\, \partial_r \left[a_0 \tilde{A}_t + \frac{Q_h^2 \tilde{A}_t^2}{4\pi} + \tilde{N}^r_t \right], \tag{37}$$

where d_h stands the value of the covariant flux of energy momentum tensor at the event horizon. The $\nu = t$ component of the Ward identity can be written as

$$\begin{aligned} \nabla_\mu \tilde{T}^{\mu}_{t} = \partial_r \tilde{T}^{r}_{t} = a_0 \partial_r \tilde{A}_t(r) + \left[\partial_r \left(\frac{Q_h^2}{4\pi} \tilde{A}_t^2(r) + \tilde{N}^r_t \right) H \right] \\ + \left(\tilde{T}^{r}_{t(0)} - \tilde{T}^{\mu}_{t(H)} + \frac{Q_h^2}{4\pi} \tilde{A}_t^2 + \tilde{N}^r_t \right) \delta(r - r_+ - \varepsilon). \end{aligned} \tag{38}$$

The first term in the right is the classical effect arising from the Lorenz force, the second term has to be canceled by the quantum effects of the classically irrelevant ingoing modes that contribute to the energy momentum tensor $-(Q_h^2 \tilde{A}_t^2(r)/4\pi + \tilde{N}^r_t)$. Thus, to save underlying diffeomorphism invariance at the horizon of the Kerr-Newman-Kasuya black hole at the quantum level, the properties of the delta function should impose its coefficients satisfy

$$d_0 = d_h + \frac{Q_h^2}{4\pi} \tilde{A}_t^2(r_+) - \tilde{N}^r_t(r_+). \tag{39}$$

After the covariant boundary condition that requires the covariant energy momentum tensor $\tilde{T}^{r}_{t(H)}$ to vanish at the horizon is imposed, the total covariant compensating flux of energy momentum tensor, which can hold the general coordinate invariance at the event horizon of the Kerr-Newman-Kasuya black hole, can be expressed as

$$d_0 = \frac{Q_h^2}{4\pi} \tilde{A}_t^2(r_+) - \tilde{N}^r_t(r_+) = \frac{Q_h^2 (Q_h r_+)^2}{4\pi (r_+^2 + a^2)^2} + \frac{\pi T_+^2}{12}, \tag{40}$$

where $\kappa = \frac{1}{2} f'(r)|_{r_+} = 2\pi T_+ = (r_+ - r_-)/2(r_+^2 + a_+^2)$ is the surface gravity of the black hole and T_+ is the Hawking temperature.

Now, we focus on exploring the relation between the compensating fluxes and those of Hawking radiation. The Planck distributions of the blackbody with chemical potential take the form as

$$B_{\pm Q_h}(\varpi) = 1\Big/\left(\exp\left(\frac{\varpi \mp Q_h \tilde{A}_t(r_+)}{T_+}\right) - 1\right),$$
$$F_{\pm Q_h}(\varpi) = 1\Big/\left(\exp\left(\frac{\varpi \mp Q_h \tilde{A}_t(r_+)}{T_+}\right) + 1\right), \tag{41}$$

for bosons and fermions respectively, where $\varpi = \omega - m\Omega_+$ is the energy that an observer hold in the dragging coordinate frame while Ω_+ is the dragging angular velocity at the event horizon and m is the azimuthal quantum number. For the case of bosons in the zero temperature limit, the absorption coefficient may become negative, leading to the effect known as super-radiance. While for fermions, occupations for these low frequency modes become one at zero temperature, which leads to a nonzero flux even at the extreme case [11]. To keep things simple, we therefore focus on the fermions case and find

$$\int_0^\infty \frac{Q_h}{2\pi}[N_{Q_h}(\varpi) - N_{-Q_h}(\varpi)]d\varpi = \frac{Q_h^3 r_+}{2\pi(r_+^2 + a^2)}, \tag{42}$$

$$\int_0^\infty \frac{\omega}{2\pi}[N_{Q_h}(\varpi) + N_{-Q_h}(\varpi)]d\varpi = \frac{Q_h^2(Q_h r_+)^2}{4\pi(r_+^2 + a^2)^2} + \frac{\pi T_+^2}{12}. \tag{43}$$

Comparing (32) and (40), which are obtained from the conditions to restore the gauge and general coordinate invariance, with (42) and (43), we find the fluxes of energy momentum tensor together with charge flow precisely equal to those of a $(1+1)$-dimensional blackbody at the Hawking temperature with an appropriate chemical potential. Surely for the bosons distribution, we also can get the same result.

5 Concluding Remarks

For the effective theory in the charged and magnetized black hole space time, the major problem is how to treat the gauge flux of magnetic charge. In this paper, we introduced an equivalent charge of the radiant electric and magnetic charges by reconstructing the electromagnetic field tensor of the Kerr-Newman-Kasuya black hole. By this, the background space time is fully similar to the Kerr-Newman black hole and the anomaly cancellation method is applicable. In the dragging coordinate frame, we hence only considered one gauge anomaly and one gravitational anomaly.

In fact, we can verify our work from the first law of black hole thermodynamics. It's well known that the general differential form of the first law of black hole thermodynamics takes the form as

$$dS_{BH} = \frac{1}{T}dM - \frac{\Omega}{T}dJ - \frac{V_e}{T}dQ_e - \frac{V_g}{T}dQ_g. \tag{44}$$

As for the Kerr-Newman-Kasuya black hole

$$V_e = \frac{Q_e r_+}{r_+^2 + a^2}, \qquad V_g = \frac{Q_g r_+}{r_+^2 + a^2}. \tag{45}$$

Taking (7) and (8) into account, (43) can be easily rewritten as

$$dS_{BH} = \frac{1}{T}dM - \frac{\Omega}{T}dJ - V_h dQ_h, \tag{46}$$

this is similar to that only with electric charge. Obviously, our work is consistent with the initial viewpoint of Robinson and Wilczek that the compensating fluxes, which are required to cancel the gauge anomaly and gravitational anomaly at the quantum level to restore the underlying gauge covariance and general coordinate invariance at the event horizon, are equivalent to those of a 2-dimensional blackbody at the Hawking temperature. Our work can also be applied to the other black holes with electric charge and magnetic charge.

It should also be referred that in this paper we only considered the scalar field, as a matter of fact, to get the full information of Hawking radiation from the viewpoint of anomaly, one should also consider the case of vector field and spin field. Recently references [31] and [32] respectively discussed Hawking radiation of vector particles and spin particles from the anomaly point of view, both results of them supported the picture that Hawking radiation can be regarded as an anomaly eliminator at horizons. Thus our skill can also be extended to these cases.

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